SUP085754

Module code: M337 Complex analysis

Module Essentials

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M337 Guide

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Introduction

Welcome to M337! Complex analysis is a rich topic that is of foundational importance in mathematics and physics. It plays a key role in disciplines such as aerodynamics, control theory, quantum mechanics and statistical physics, and it is an integral part of advanced mathematical subjects such as analytic number theory, functional analysis and Riemann surface theory. This module covers the standard topics of a typical course in complex analysis, as well as some exciting additional topics, including the Riemann zeta function, fluid flows and the Mandelbrot set.

M337 leads you through a number of advanced mathematical concepts, bringing together ideas from analysis, algebra and geometry. You should expect to find it challenging, and you should expect to have to work hard to master the techniques that you meet. After all, as you will see from the historical narrative within the texts, some of the greatest mathematicians of all time struggled with the the subtleties of complex analysis, and it was only through hundreds of years of work that the subject has reached its present polished state.

In completing the module you will come to appreciate the fascinating network of interrelated concepts that make complex analysis such a powerful subject. It will consolidate many of the mathematical ideas and methods that you have learned in earlier modules, and it will set you in good stead for tackling further fields of study in mathematics, engineering and physics.

We wish you all the best in your studies of M337!

The M337 module team

Ian Short (chair), Andrew Potter, Phil Rippon, Hayley Ryder and Angela Russell (manager)

Module components

Study texts

There are four texts, comprising thirteen units of study.

Book A: Complex numbers and functions

The first book, made up of four units, provides an introduction to complex numbers and complex functions. It describes many basic properties of complex functions, including the property of being analytic.

Book B: Integration of complex functions

The second book, which also has four units, contains Cauchy's Theorem, one of the main results in complex analysis. This leads to several representations of analytic functions by means of contour integrals and power series.

Book C: Geometric methods in complex analysis

The third book, of three units, introduces Cauchy's Residue Theorem and uses it to evaluate improper integrals and infinite series, and to locate zeros of analytic functions. Here you will meet some of the other advanced results in complex analysis, and you will also be introduced to the subject of conformal mappings.

Book D: Applications of complex analysis

The final book consists of two units, on two distinct topics, namely fluid flows and the Mandelbrot set. In Unit D1 you will see how techniques from complex analysis can be applied to create a physical model of the flow of a fluid, subject to certain constraints. Then in Unit D2 you will learn about modern research in a branch of complex analysis known as complex dynamics.

You may notice that the module texts are fairly long! This is because they provide detailed background material for each topic, not assuming that you will have met the key concepts before. In addition, the texts are illustrated with over 1000 figures, to help your understanding. You will also find a large number of exercises, with subsections of 'Further exercises' at the end of each section. You should attempt as many exercises as you have time for.

Handbook

The Handbook summarises the main definitions, results and methods from the module. It is a good idea to start using the Handbook from the beginning of your studies, so that you familiarise yourself with its contents.

You can and should take the printed copy of the Handbook into the examination with you (but no other module materials). This version of the Handbook must be the printed copy that was sent to you, rather than a version that you have downloaded and printed yourself.

The Handbook that you take into the examination is allowed to have 'basic annotation', which is specifed by University regulations as follows.

> The text as printed may be supplemented by handwritten highlights (for example by a highlighter pen or by ringing, underlining or sidelining), and by corrected typographical errors. The addition of comments, marginal notes, notes in the blank spaces at the end of paragraphs and pages or on fly-leaves is not permitted.

To reference an item from the Handbook you can write, for example,

> HB A2 4.12, p23,

to refer to item 12 in Section 4 of the Unit A2 part of the Handbook, on page 23.

Other formats for referencing are acceptable, provided that they are clear and accurate.

Website

The module website includes the study planner, tutor-marked assignments (TMAs) and the specimen exam papers (and solutions). These are available only via the module website. You will also find electronic versions of the printed study texts and Handbook.

It is important that you check the website frequently for news items relating to the module, and any errata that may arise.

The module website gives you access to the forums, which you can use to communicate with other students and with the M337 team. Access to online rooms (if your tutorials are electronic) is also through the module website.

Assessment

There are two aspects to the assessment of M337:

- continuous assessment, in the form of four tutor-marked assignments (TMAs)
- a three-hour examination.

Continuous assessment

There is one TMA per book. The final TMA, for Book D, also includes some revision questions for Books A, B and C.

Your overall continuous assessment score (OCAS) is determined by taking the average of your four TMA scores.

You can submit your TMA solutions either by post or online. If you submit online, then you must submit your solutions as a single PDF (a Word document or a set of several PDFs will not be accepted). Please read the instructions on the module website carefully before submitting your work.

Examination

Your overall examination score (OES) is determined by a three-hour examination at the end of the module. To prepare for the examination, we recommend that you consult the M337 specimen papers and the previous M337 examinations, and be prepared for some limited variation in the style and content of questions.

You may notice that the exam format for the 2018 exam and for earlier exams is different to the current format; in particular, previously there were eight Part 1 questions, whereas now there are six Part 1 questions. This change was made to allow more thinking time in the examination and to prevent examinees from being rushed.

You should take the Handbook into the examination, but no other module materials are allowed. Remember that **only basic annotation of the Handbook is permitted**, as specified in the Module components section.

Calculators are not needed in the examination, and are specifically prohibited.

Your overall assessment score

Your overall score for the module will be calculated as:

> 0.2 x your continuous assessment score + 0.8 x your examination score.

For example, if your continuous assessment score (the average of your four TMA scores) is 75%, and your examination score is 50%, then your overall score for the module is 0.2 x 75% + 0.8 x 50% = 55%.

It is essential that you engage fully with the TMAs, as these are an important part of the module. They develop your learning and understanding of the topics covered and provide you with tutor feedback on your work. **Taking time to work through the TMA content**

and the feedback you receive in detail is vital for successful examination preparation.

The contribution of each assessment score to the overall score is given in the table below.

Assessment	Contribution to overall score
TMA 01	5%
TMA 02	5%
TMA 03	5%
TMA 04	5%
Examination	80%

Based on your overall score you will be awarded one of the following results:

- Pass 1 / Distinction
- Pass 2
- Pass 3
- Pass 4 / Pass
- Fail.

Normally, for a Pass 4 result you must achieve an overall score of at least 40%.

Support for your studies

It is important that you try to keep to schedule. Each assignment cut-off date is soon after the end of the last study week for the relevant units. We recommend that you try to finish the assignment questions relating to each part of a unit as soon as you finish that part. This will ensure that you stay on top of the material, and do not have to rush shortly before the cut-off date.

Your tutor

Your tutor is there to help you to understand the ideas in M337, and he or she will provide comments and feedback on your written assignments. You are advised to go through each marked assignment in detail, and to take note of the comments written by your tutor; they will help you to avoid similar errors in later TMAs and in the exam. Try to attend tutorials – either face-to-face or online – where you will have the opportunity to talk to your tutor and other students directly.

Your fellow students

One of the best ways of learning is by talking about your work with fellow students. A convenient way to keep in touch with other students is to use the M337 forums, which are moderated by members of the M337 module team.

Other support

You are not expected to study alone. Support is available from your tutor, through tutorials, and through the module website. If you experience difficulties that are not directly related to the content of M337, then you are welcome to contact your Student Support Team (see your StudentHome page for details).

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Accessibility for Mathematics and Statistics modules

Introduction

The Open University provides a range of support services for disabled students. You may not think of yourself as disabled but if you have a long term health condition, specific learning difficulty (such as dyslexia) or mental health difficulty then you are entitled to support. We use the single term ‘disabled’ or ‘disability’ to cover all these conditions. You can find more information about the services that are available by going to the Supporting students with disabilities website which links through to Overcoming Barriers to study if you have a disability or health condition. For example, you may need to have module materials in an alternative format, extra tutorial support or flexibility with some assignment deadlines because of your disability or condition.

If you live in the UK, you may be able to get a grant called the Disabled Students’ Allowance (DSA). This aims to cover the extra costs of studying in higher education – for example, the need for specialist equipment that might arise because of your disability, medical condition or specific learning difficulty. Further information on the DSA is available on the who provides support and adjustments website, and you can also visit Disabled student support. If you would like to discuss your needs, before registering as a student, please contact us. Any sensitive information will be treated confidentially.

If you are already an Open University student, you can also find more information on the Disabled student support website. Once you have read this guide, if you think that you will need additional support, you can submit a Disability Support Form outlining your requirements and then discuss the help that you need with an adviser; if you will need help to complete the form over the phone, please contact us.

You can also obtain advice and information from your Student Support Team.

> Please contact us as soon as possible to discuss your requirements, as some support services can take several months to arrange.

This guide outlines some of the general accessibility issues that apply to distance learning of mathematics and statistics. Many of the issues discussed apply to both mathematics and statistics, and you can assume that references to mathematics also apply to statistics. This guide includes advice on:

- using the module materials
- using software that may improve accessibility
- writing mathematics and statistics
- assessment.

The guide does not provide advice on the accessibility of individual modules, or additional or alternative resources that they may provide. This information may be provided in the module description and either on the module website or in the module guide.

General information on the accessibility of the OU’s online resources is available in the accessibility section of The Open University's Computing Guide .

Note that some of the website links in this Accessibility Guide are available only to registered students.

Studying mathematics and statistics

Each mathematics or statistics module involves using a wide range of resources, including some or all of the following:

- module website
- printed units
- scientific calculator
- computer software
- audio, video and tutorial clips (screencasts).

In addition, you will have the opportunity to join in discussions with your tutor and other students, in tutorials and through online forums and email. The tutorials may take place at a study centre or using the University's *Online Rooms* environment, which combine voice, text and images.

By its nature, mathematics is a very visual subject, and most mathematics and statistics modules contain considerable amounts of mathematical notation, graphs and diagrams that you will need to be able to interpret and produce. You may also need to use the computer and mouse to manipulate and enter data. You may need to consider if you are likely to experience problems with activities and assessment questions that involve using your computer, or have high graphical content, particularly if you have difficulties reading print or using a computer for any length of time.

The Open University is committed to making every reasonable effort to make its teaching accessible and to use accessible resources, however some material that is core for a module may not be easily accessible, even if you use assistive technology. You may need a non-medical helper to assist you. If you have any accessibility problems or comments that would help us improve a particular module, please ask your study advisor or tutor to let the module team know. Information about accessibility issues that arise during the life of the module will be posted on the module website.

The module websites

The module websites provide access to a range of resources. These include the Study planner, module resources, any interactive activities and assessment materials. Each website has been checked for accessibility and has an 'Accessibility' link at the top of each page. This provides a link to the OU Accessibility site which has advice on increasing the text size and changing your browser settings, such as colours and fonts. It also includes a link to the accessibility section of the Computing Guide and some useful external links.

The Resources tab may contain a section on Accessibility, which may include figure descriptions of the diagrams and images in the module, and written transcripts of the videos and tutorial clips (screencasts). It may also contain other accessibility resources related to the module. Please refer to the individual module guides for information about this.

Searchable PDF versions of printed material are available in the Resources section of the module website. However, mathematical and statistical content in PDF files is unlikely to be accessible using a screenreader, and older modules may have PDF files that are not searchable. You may need additional help to read these documents.

Modules in mathematics and statistics may require you to spend some or a lot of time online, for example viewing screencasts, completing Interactive Computer Marked Assignments (iCMAs), attending online tutorials or engaging with forums. If you have any accessibility problems relating to this, please ask your study advisor or tutor to let the module team know.

Printed units

The latest information on alternative formats for printed units is available from the Services for Disabled Students website.

Text descriptions are provided for all printed diagrams, charts and graphs that require them. In some cases, you may find it helpful to use the PDF file and enlarge the diagram or graph on your computer screen. An audio graphing calculator can also be used for some activities.

There is free software such as MathTrax that can provide an audio version of some graphs and datasets, and can be used alongside a screenreader. This type of software can be useful if you have difficulty reading text, either printed or on screen.

MathTrax can be downloaded free from the MathTrax homepage.

Notes are available on using MathTrax to plot datasets and graph equations. Further information on MathTrax is also available.

If the effects of your disability make it difficult for you to produce written mathematics, graphs or diagrams, you may like to consider using Efofex, particularly if you are studying Y033 or MU123. This is available free to disabled students under the EmPower programme. Further details are available on the EmPower website or by emailing info@efofex.com for further information.

Scientific calculators

You may need a scientific calculator for your module. Any model that meets the module specification can be used, including computer-based calculators, talking calculators and those with large keys and displays. You can request the loan of a talking scientific calculator from the borrowing small equipment webpage. You can also use the calculator that comes with your computer, if you wish. On the Windows calculator, for example, the menu contains an option to switch the calculator to scientific mode.

You should note that only certain types of calculators are permitted in examinations; further details are available at Calculators permitted in examinations in mathematics and statistics.

Emulators for some types of calculators are available. These enable you to use a calculator on your computer screen and to enlarge the screen and keyboard. An emulator for the Casio fx-82 and Casio fx-85 is available for Windows 7, 8 and 10 from ClassWiz Emulator Subscription v2.00 (90 Day Trial) for Windows. This link includes emulators for many calculators, so please check that you have downloaded and installed a version that meets the exam requirements (for example, the fx-82/85/350EX emulator).

Computer resources

All computer resources have been checked carefully for accessibility, and they can be operated using the keyboard as well as the mouse, unless stated otherwise in the module description. Some components also have their own shortcut keys. For some dynamic computer resources, you may wish to consider the services of a non-medical helper, even if you have assistive technology.

A variety of software applications, some specifically for OU modules and some for more general use, are available for students to download from the Software Downloads section of the Computing Guide.

The Java Access Bridge, which enables Java applications and screenreaders to work together on Windows PCs, is installed as part of the standard Java package; users of assistive technology with Java applications should ensure that the Java Access Bridge is enabled; explicit instructions are given in the Additional software required section of the Computing Guide.

Audio, video and tutorial clips (screencasts)

All audio, video and tutorial clips (screencasts) have transcripts, subtitles and closed captions or alternative versions, unless stated otherwise in the module guide.

Tutorials

Tutorials may be offered either face-to-face or online using the university's conferencing system, *Online Rooms*. Please discuss your requirements with your Student Support Team or with your tutor, particularly if you are deaf or hard of hearing, or use assistive technology such as a screen reader, or if you need particular colour settings.

Writing mathematics using a computer or laptop

This section is aimed at those of you who either need or wish to use a computer or touchscreen to prepare your assignments. Typesetting mathematics is a time-consuming process, but learning how to do it may be a good investment if you are intending to study a lot of mathematics. The Mathematics and Statistics Study website contains information on typing mathematical notation by using either Microsoft Word or the mathematical typesetting language LaTeX. A link to this guidance is under the Skills tab.

Typesetting, even of elementary mathematics, can be tricky. For screenreader users, there are specific issues in that although you may be able to type in maths expressions, the screenreader may not be able to read them back to you. The essential reason why typesetting mathematics is difficult is that written mathematics is two-dimensional in nature. Objects like fractions and matrices have a two-dimensional format and are sometimes stored as images rather than text. This is why screenreaders have difficulty. The good news is that technology is improving all the time and there is software that can help you.

If you use a screenreader and intend to study mathematics beyond Level 1, then you are advised to learn how to use LaTeX, a freely available mathematical typesetting package. This system is long established and it is used by professional mathematicians. The mathematical expressions are typed in a straightforward (plain text) way; experienced blind or/and partially sighted people/students users are likely to read mathematics, as well as write it, in this format. There is further information about Braille and LaTeX on the latex-access sourceforge website.

If you are going to work fully in LaTeX, you might wish to investigate editing software called EdSharp. This text editor has been designed with screenreaders in mind and incorporates facilities for editing LaTeX, HTML and other coding languages.

If you are studying mathematics as part of subject area where Microsoft Word is the normal document software, then you have two main options for entering mathematical content: you may either use Word's inbuilt equation editor, or you may use MathType. If you wish to use Word's inbuilt equation editor, you may like to access this video on Getting started with effective entry of equations in Word. If you obtain version 6.5 or higher of MathType, then you will be able to type in your mathematics using LaTeX notation, but manage the rest of the document in the usual way. You can then toggle between the LaTeX and the maths display by pressing Alt and the backslash key (\) together from within Microsoft Word. This should help you to check your work. There are some further details on the access2science Authoring Math website.

If you decide to use MathType for both your assignments and discussions with your tutor, please check that your tutor can see what you have written. If your tutor does not have MathType installed, then they may not be able to see your mathematical statements. One option is to save your document as a PDF file, so that your tutor can read it easily; this is a reliable method and is recommended.

Some assistive technology, such as ZoomText works best when used with equations produced using Microsoft Word's equation editor. When increasing the font size in the Word document, equations produced in Word format will reflow while those produced with MathType will not.

Some students have found using the commercial software MathTalk helpful for dictating mathematical statements. However, if you decide to use Mathtalk for any assessments, then you must turn off the computer algebra engine and not use the Evaluate, Solve, Integrate or other similar commands. To turn off the engine in MathTalk, please click on Tools, then Computation Setup and then select None in the `Current Engine' drop-down box. The YouTube video titled turning off engine in scientific notebook illustrates how to do this.

Assessment

Assessment in the form of Tutor-Marked Assignments (TMAs) can be submitted by posting a paper copy to your tutor, or electronically submitted using The Open University eTMA system. TMA questions may be in PDF format or accessed directly from the website. In places where graphs or diagrams are used, descriptions will be supplied. You should be able to complete the TMAs successfully with appropriate support from your tutor or helper.

iCMAs are submitted online and may have some interactive content. If you are unable to access the internet (for example, if you are a Student in a Secure environment) or are unable to use the iCMA system for other exceptional reasons, please contact your Student Support Team.

The examinable component for your module may be in the form of an end-of-module assessment (EMA) or an exam. If you have already notified us about a long term health condition, specific learning difficulty or mental health difficulty, you will be contacted by your Student Support Team to discuss any requirements that you may have for the exam.

If you are unable to submit all your assignments or have extra difficulties during the EMA or the exam, you are advised to notify the Exam Board to explain your circumstances. You can do this online by filling in a special circumstances form. If you need further help with the assessment on your module, please contact your tutor.

Summary of web links

The Accessibility section of the Computing Guide has general advice on accessibility for Open University modules, including information on accessibility of equations on module websites. Some external sites also have useful information; for example, Microsoft accessibility has some useful pages on how to access computers and on Windows keyboard shortcuts for accessibility. Similar information is available for Apple computers.

The list below summarises the main websites mentioned in this guide; you can access the university links from your StudentHome page.

We wish you every success with your study of mathematics and statistics. If you do experience any difficulties during your studies, please contact your tutor, who will be able to help you with the assistance of other OU staff in Study Support and Services for Disabled Students.

Useful web links

Description	Web address
Accessibility section of the Computing Guide	https://learn1.open.ac.uk/course/view.php?id=100132&cmid=12929
Accessibility - using a computer for OU study	https://help.open.ac.uk/accessibility-using-a-computer-for-ou-study
Additional software required section of the Computing Guide	https://learn1.open.ac.uk/mod/oucontent/view.php?id=8329&section=5
Borrowing small equipment	https://help.open.ac.uk/borrowing-small-equipment
Calculators permitted in examinations in mathematics and statistics	https://learn2.open.ac.uk/mod/oucontent/view.php?id=1187836

ClassWiz Emulator Subscription v2.00 (90 Day Trial) for Windows	https://edu.casio.com/softwarelicense/index.php#col2
Contact us	http://www.openuniversity.edu/contact-us
Disability Support Form	https://help.open.ac.uk/disability-support-form
Disabled student support	http://www2.open.ac.uk/students/help/topic/disability
EdSharp	https://github.com/EmpowermentZone/EdSharp
Efofex	http://efofex.com/
Getting started with effective entry of equations in Word	http://www.mathcentre.ac.uk/bathmash/Word/index.html
LaTeX − Braille converter	http://latex-access.sourceforge.net
LaTeX in MathType	www.access2science.com/jagqn/WordLatex.html
Mathematics and statistics	https://learn2.open.ac.uk/site/s-maths
MathTalk	http://www.mathtalk.com
MathTrax	https://prime.jsc.nasa.gov/mathtrax/
MathTrax − drawing graphs	http://prime.jsc.nasa.gov/mathtrax/SHOWME/equations.htm
MathTrax − plotting datasets	http://prime.jsc.nasa.gov/mathtrax/SHOWME/data.htm
MathType	www.dessci.com/en/products/mathtype
Module website accessibility	https://help.open.ac.uk/accessibility-using-a-computer-for-ou-study
Overcoming Barriers to study if you have a disability or health condition	http://www2.open.ac.uk/students/help/overcoming-barriers-to-study-if-you-have-a-disability-or-health-condition
Software downloads	https://learn1.open.ac.uk/mod/oucontent/view.php?id=12658
Special circumstances	www.open.ac.uk/assessment/requests/special-circumstances.php
Student support team	https://help.open.ac.uk/your-contacts
Supporting students with disabilities	http://www.open.ac.uk/courses/do-it/disability
Turning off engine in scientific notebook	https://www.youtube.com/watch?v=ch0PU4QXlFc
Typing Mathematical notation	https://learn2.open.ac.uk/course/view.php?id=206217&cmid=1172763
ZoomText	https://www.zoomtext.com/

‹ Accessibility for Mathematics and Statistics modules

Administration

M337 examination guidance

Preparing for the examination

To prepare for the M337 examination, we recommend that you carry out the following tasks.

1. Decide on a realistic revision and exam strategy, remembering that there is choice in Part 2 of the exam.
2. Work through all three specimen examination papers thoroughly. You may like to reserve one exam as a mock exam that you carry out under self-imposed exam conditions.
3. Review the TMA questions and your tutor's feedback carefully, making sure you understand solutions to all the questions.
4. Attempt as many of the exercises in the M337 module texts as possible.
5. You may also choose to attempt questions from previous years' M337 exams. Remember that the content of the module has changed slightly, as has the exam format. Nonetheless, all the past questions should provide helpful practice for this year's exam.

For support with these tasks, contact your tutors, or post questions on the forums.

Examination technique

Here are suggestions to improve your examination performance.

1. Begin the exam with a question that you feel more comfortable with, to help you settle into a rhythm.
2. If you get stuck on a question and find that you are unable to make progress, then put it aside, move on to another question and return to it later.
3. Remember that it is normal and common to feel nervous in an examination. It has happened to everyone, including the examiners. Should you become unsettled, then search for a question you feel comfortable with, and focus on working steadily through that.
4. Read all the questions that you attempt carefully, answering all parts of them (remember you can guess if you do not know an answer).
5. Attempt *all* questions from Part 1, and attempt two out of three questions from Part 2. You can of course attempt all three questions from Part 2 (in which case your two highest-scoring solutions will count); however, you will make more efficient use of time if you try only two questions from Part 2.
6. If you have time left at the end, then *check your answers*. It is hard to avoid some mistakes. Find them!
7. If you cannot complete part of a question, then don't give up on it completely. Questions often have 'restart points' – these are later parts of a question designed so that they can still be attempted by someone who has not managed an earlier part. Try the later parts; try or guess as many parts as possible to maximise your marks.
8. Sometimes the latter parts of a question may give away a hint for the earlier parts of the question – perhaps, for example, there is a formula later on in the question that helps with the first part of the question. So please read the whole of each question you attempt.

9. Work hard for the duration of the examination, checking and improving your work if you have spare time.
10. Should you get into a tangle involving awkward mathematical expressions, then this may be because of an earlier algebraic slip. Have a look.
11. You can present short answers, but don't make them *too* short. Remember that the examiners have a precise marks scheme, so they need to see working in order to award you full marks for a question.
12. On the other hand, don't make your answers too long. The specimen examinations are provided with model solutions to guide you on ideal length. (However, your solutions do *not* have to be of exactly that standard and length in order to get full marks.)
13. To finish each argument that you present in your solutions, you could consider writing a phrase such as 'as required' or 'so the result follows'.

Handbook referencing

1. As a guide, you should reference any significant theorem that you use, but you don't need to reference minor results. For example, you should reference Rouché's Theorem if you use it, but you needn't reference the property $|\overline{z}| = |z|$ if you need it.
2. If you are in doubt about whether to reference a result or not, then it's advisable to put a reference in, just in case.
3. Use any clear referencing style that you wish. For example, you could refer to Rouché's Theorem or, alternatively, you could refer to Unit C2, item 2.6, page 67. In this case the former is preferable because it is clearer to read, but either referencing style is acceptable.
4. When applying a result from the Handbook, *make sure you check that the hypotheses are satisfied*. For example, Rouché's Theorem requires two analytic functions f and g on a simply connected region $\mathcal{R}$, with various other properties. Make sure you specify f and g, describe $\mathcal{R}$, and so on.

Mathematical tips and common slip-ups

1. Often questions ask you to give your answer in a particular form (such as Cartesian form or polar form). Try to make sure you do so!
2. Remember that $1/i = -i$ (don't forget the minus sign).
3. Inequalities should involve only real numbers; you cannot write $2i < 3$, say, or $z < 3$, where z is some unknown complex number.
4. A real integral should have a real value.
5. The imaginary part of a complex number does not involve the symbol i; for example, the imaginary part of $-1 + 3i$ is 3 rather than $3i$.
6. Make sure that you apply the Strategy for determining principal arguments correctly, using $\phi = \tan^{-1}(|y|/|x|)$ (not $\phi = \tan^{-1}(y/x)$), and use the right formula relating θ and ϕ.
7. When manipulating trigonometric and hyperbolic expressions, it is often simplest to use trigonometric and hyperbolic identities rather than writing the expressions in terms of sums and products of exponential functions.
8. Remember that $\operatorname{Arg} z$ lies in the interval $(-\pi, \pi]$, where $z \neq 0$. Since $\operatorname{Log} z = \log|z| + i\operatorname{Arg} z$, it follows that the imaginary part of $\operatorname{Log} z$ lies in the interval $(-\pi, \pi]$.
9. Avoid confusing $\log$ and Log. You should only write $\log x$ when x is a real positive number.
10. A region is both open and connected. If the region is specified in set notation, then typically the specification will involve strict inequality symbols $<$ and $>$; not $=$, $\leq$ and $\geq$.
11. To find an upper bound for the expression $|f(z)|/|g(z)|$ you need to combine an upper bound for $|f(z)|$ with a *lower* bound for $|g(z)|$. For example, if $|f(z)| < r$ and $|g(z)| > s$, then $|f(z)|/|g(z)| < r/s$.

12. Cauchy's Integral Formula involves an expression of the form
$$\int_\Gamma \frac{f(z)}{z-\alpha}\,dz.$$
If you have an expression such as
$$\int_\Gamma \frac{g(z)}{3z-2}\,dz,$$
then you first need to write it in the form
$$\int_\Gamma \frac{g(z)/3}{z-2/3}\,dz$$
before you apply Cauchy's Integral Formula (with $f(z)=g(z)/3$ and $\alpha=2/3$). A similar warning applies to the Cover-up Rule.
13. Don't mix up a statement and its converse. For example, the statement about the Mandelbrot set M that '$|c+1|<\frac{1}{4} \Longrightarrow c\in M$' is true, but the converse statement '$c\in M \Longrightarrow |c+1|<\frac{1}{4}$' is false (because $c=0$ lies in M but $|0+1|>\frac{1}{4}$).
14. The complex conjugate of the expression
$$\frac{1}{\overline{z}+i}$$
is
$$\frac{1}{z-i}$$
(not $1/(z+i)$). Don't forget to conjugate the i. Similar comments apply to other, similar expressions!

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Feedback

Faculty of Science, Technology, Engineering and Mathematics
M337 Complex analysis

M337

Diagnostic quiz

Am I ready to start *Complex analysis* (M337)?

The aim of this diagnostic quiz is to help you assess how well prepared you are for M337, and to identify topics that you should revise or study before beginning the module. It is important that you are well acquainted with these topics beforehand, because parts of them will be assumed in M337, sometimes without comment. You should prepare well so that you can give your full attention to the new material that you will meet in M337.

M337 is a third-level mathematics module and as such aims to develop a higher level of mathematical maturity than that you may have required in studying second-level Open University modules in mathematics. A good pass (grade 1 or 2) in a module such as M208 *Pure mathematics*, MST210 *Mathematical methods, models and modelling* or MST224 *Mathematical methods* will serve as a solid foundation for developing these new skills.

The quiz consists of twenty-five questions split over six sections, with worked solutions at the end. The first five sections of the quiz (Algebra, Basic functions, Sketching sets and curves, Sequences and series, and Differentiation and integration) test your knowledge and facility with the fundamental mathematics that will be assumed throughout M337. The concepts illustrated in these sections will be used in the deeper parts of M337, so it is important that you are able to cope with these concepts in order to get to grips with the module. The final section (Analysis) tests your understanding of some slightly more advanced concepts. These concepts should be familiar if you have studied M208. If you have not studied M208, then some aspects of these questions may be new to you, in which case you may have to brush up on a few topics before or at the start of your studies of M337.

Solutions are provided at the end of the quiz (from page 6), along with advice and guidance (labelled 'Feedback') to help you decide whether you are ready for M337, and, if not, what you might do about it. Please read all the advice, even if you don't look at all the solutions. We suggest that you complete all twenty-five questions before reading any solutions.

If you find that you can work your way though the whole quiz within two hours, with only the occasional need to look at material you've previously studied, then you should consider yourself well prepared for M337. It is more likely that you will find there are one or two topics on which you

are a little rusty, in which case we suggest you refer to some of the resources suggested below.

After working through the quiz, you may still be unsure whether M337 is the right module for you; in that case you should visit the Study Support section of StudentHome, or consult your Student Support Team, for further advice.

Resources to help you revise for M337

All the topics of this quiz are covered in detail in M208, so if you have studied that module, then you can revise by referring to the M208 module materials.

You may have studied MST210 or MST224 already, in which case you should revise those aspects of the module materials that are relevant to M337.
Many of the basic mathematical skills needed for M337 (and needed to tackle the quiz) can be found in the level one modules MST124 *Essential mathematics 1* and MST125 *Essential mathematics 2*.

Questions

You should not use a calculator to attempt the quiz. In M337 we use exact real values such as $\sqrt{2}$, π and e rather than decimals. (Note that the use of calculators is not permitted in the end-of-module examination for M337.)

Algebra

1. Simplify $3(2c(a+b)+4)-6(a(b+c)+2)$.
2. Determine which of the following expressions is equal to $\dfrac{1}{a}-\dfrac{1}{b}$.
   (a) $\dfrac{1}{a-b}$ (b) $\dfrac{b-a}{ab}$ (c) $\dfrac{a-b}{ab}$ (d) $\dfrac{a+b}{a-b}$ (e) $\dfrac{b-a}{a+b}$
3. Simplify $\dfrac{1}{\sqrt{2}+1}$.
4. Determine which of the following expressions is equal to $(a-b)^3$.
   (a) a^3-b^3
   (b) $a^3-3a^2b+3ab^2-b^3$
   (c) $a^3+3a^2b-3ab^2-b^3$
   (d) $a^3-3a^2b+3ab^2+b^3$
   (e) $a^3+3a^2b-3ab^2+b^3$
5. Factorise x^3-5x^2+6x.
6. Find the set of all values of x for which $x(x-3)>0$.
7. Write $\dfrac{1}{x(x-3)}$ in partial fractions.

Basic functions

8. Complete the following table.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\sin\theta$	0		$1/\sqrt{2}$		1				
$\cos\theta$		$\sqrt{3}/2$				$-1/2$			
$\tan\theta$	0				*				

9. Show that $\sin^2(2\theta) = 4\cos^2\theta(1-\cos^2\theta)$.

10. Find *all* values of $\theta \in [0, 2\pi)$ for which $\cos(2\theta) = -1/2$.

11. Simplify each of the following expressions. (Note that in some other modules $\log_e$ is written as ln.)

    (a) $(e^{-1})^{-1}$ (b) $\dfrac{e^{2x} \times e^{3x+1}}{e^x}$ (c) $\frac{1}{3}\log_e 8 - \frac{1}{2}\log_e 4$ (d) $e^{-\log_e 2}$

12. Find *all* real solutions of $e^{2x} + 2e^x - 3 = 0$.

    Hint: Let $y = e^x$ and solve the corresponding quadratic equation in y.

Sketching sets and curves

13. Sketch the sets $\mathbb{R} - \{n/2 : n \in \mathbb{Z}\}$ and $\{x \in \mathbb{R} : |x-2| < 3\}$.

14. Show on a diagram the region of the plane consisting of those points for which

    $$x + y > 0 \quad \text{and} \quad x^2 + y^2 \leq 1,$$

    being careful to distinguish between those points that lie within the region and those that lie outside the region.

15. Sketch the curves given by the following equations.

    (a) $x^2 + y^2 - 4x + 2y + 1 = 0$

    (b) $x^2 + 4y^2 = 36$

    (c) $9x^2 - y^2 = 1$

Sequences and series

16. Find $\displaystyle\sum_{n=0}^{\infty} 5^{-n}$.

17. Which of the following series converge? Briefly explain your reasoning.

    (a) $1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{n} + \cdots$

    (b) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \cdots + \dfrac{(-1)^{n+1}}{n} + \cdots$

(c) $1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots + \dfrac{1}{n^2} + \cdots$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n!}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

Differentiation and integration

18. State the derivatives of the following functions.

    (a) (i) $2x^3$ (ii) $4\sqrt{x}$ (iii) $5/x$

    (b) $\cos(1 + 5x)$

    (c) $x^3 \sin(x)$

    (d) $\dfrac{\log_e(x)}{2x}, \quad x > 0$

19. Find the derivative of $f(x) = e^{2x} \cos^2(x)$ at the point $x = \pi/2$.

20. Evaluate the following integrals.

    (a) $\displaystyle\int_0^1 x^3 \, dx$

    (b) $\displaystyle\int 12x(x^2 + 1)^5 \, dx$

    (c) $\displaystyle\int_4^5 \frac{1}{x(x-3)} \, dx$

    (d) $\displaystyle\int_0^1 xe^x \, dx$

Analysis

Decide whether the following statements are true. If you believe a statement to be true, then justify why it is true. If you believe a statement to be false, then give a counterexample to demonstrate that it is false.

21. If f is a real function differentiable on the closed interval $[a, b]$, then the maximum value of f in $[a, b]$ occurs at a point $c \in [a, b]$ for which $f'(c) = 0$.

22. If f is a real function with $f(-1) = -1$ and $f(1) = 1$, then there is a point $c \in \mathbb{R}$ for which $f(c) = 0$.

23. If $\sum_{n=1}^{\infty} a_n$ is convergent then $a_n \to 0$, as $n \to \infty$.

24. Is the converse of the previous statement true?

25. Is it the case that $x < y$ if and only if $x^2 < y^2$ when

    (a) $x, y \in \mathbb{R}$, (b) both $x, y > 0$?

Solutions

Algebra

1. $$\begin{aligned} 3(2c(a+b)+4)-6(a(b+c)+2) &= 3(2ac+2bc+4)-6(ab+ac+2) \\ &= 6ac+6bc+12-6ab-6ac-12 \\ &= 6bc-6ab \\ &= 6b(c-a) \end{aligned}$$

2. **(b)**: $\dfrac{1}{a}-\dfrac{1}{b}=\dfrac{b}{ab}-\dfrac{a}{ab}=\dfrac{b-a}{ab}$

3. Observe that

$$\frac{1}{\sqrt{2}+1}=\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1.$$

This trick of multiplying by one is frequently used in complex analysis.

4. **(b)**:

$$\begin{aligned} (a-b)^3 &= (a-b)(a-b)^2 \\ &= (a-b)(a^2-2ab+b^2) \\ &= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3 \\ &= a^3-3a^2b+3ab^2-b^3 \end{aligned}$$

(Or use the binomial theorem.)

5. We observe that

$$x^3-5x^2+6x=x(x^2-5x+6),$$

and

$$x^2-5x+6=(x-2)(x-3),$$

so

$$x^3-5x^2+6x=x(x-2)(x-3).$$

6. We construct a sign table for $x(x-3)$, noting that $x(x-3)=0$ when $x=0$ and $x=3$.

	$x<0$	$0<x<3$	$x>3$
x	$-$	$+$	$+$
$(x-3)$	$-$	$-$	$+$
$x(x-3)$	$+$	$-$	$+$

Hence $x(x-3)>0$ if (and only if) either $x<0$ or $x>3$. That is, $x(x-3)>0$ if and only if x belongs to the set

$$\{x\in\mathbb{R}: x<0 \text{ or } x>3\}=(-\infty,0)\cup(3,\infty).$$

7. We need to find constants A and B such that

$$\frac{1}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

for $x \neq 0, 3$. Hence, for $x \neq 0, 3$, multiplying by $x(x-3)$ gives

$$1 = A(x-3) + Bx.$$

Comparing coefficients of x and the constant term, we find that

$$A + B = 0 \text{ and } -3A = 1.$$

Hence $A = -1/3$ and $B = 1/3$, so

$$\frac{1}{x(x-3)} = \frac{1}{3}\left(\frac{1}{x-3} - \frac{1}{x}\right).$$

(Or use the cover-up method.)

Feedback Algebraic manipulations of this type appear in modules such as MST124 and MST125. You should be confident and fluent with this algebra before you start M337.

Basic functions

8.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	-1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	*	$-\sqrt{3}$	-1	$-1/\sqrt{3}$	0

9. As $\sin(2\theta) = 2\sin\theta\cos\theta$ we see that

$$\sin^2(2\theta) = (2\sin\theta\cos\theta)^2 = 4\sin^2\theta\cos^2\theta.$$

But $\sin^2\theta + \cos^2\theta = 1$, so

$$\sin^2(2\theta) = 4(1-\cos^2\theta)\cos^2\theta = 4\cos^2\theta(1-\cos^2\theta).$$

10. The equation $\cos\phi = -1/2$ has solutions $\phi = 2\pi/3 + 2m\pi$ and $\phi = 4\pi/3 + 2n\pi$, where m and n are integers. Hence $\theta = \pi/3 + m\pi$ or $\theta = 2\pi/3 + n\pi$. In the range $[0, 2\pi)$, θ has values $\pi/3$, $2\pi/3$, $4\pi/3$ and $5\pi/3$.

11. (a) $(e^{-1})^{-1} = e^{(-1)\times(-1)} = e^1 = e$

(b) $\dfrac{e^{2x} \times e^{3x+1}}{e^x} = \dfrac{e^{2x+3x+1}}{e^x} = e^{5x+1} \times e^{-x} = e^{4x+1}$

(c) $\frac{1}{3}\log_e 8 - \frac{1}{2}\log_e 4 = \log_e 8^{1/3} - \log_e 2 = \log_e 2 - \log_e 2 = 0$

(d) $e^{-\log_e 2} = e^{\log_e(2^{-1})} = 2^{-1} = 1/2$

12. We make the substitution $y = e^x$ (and note that y must be positive). This gives

$$y^2 + 2y - 3 = 0,$$

which factorises to give

$$(y - 1)(y + 3) = 0.$$

Hence $y = 1$ or $y = -3$. However, y is positive, so we conclude that $y = 1$. Therefore any real solution x of $e^{2x} + 2e^x - 3 = 0$ satisfies $e^x = 1$; thus $x = 0$ is the only real solution.

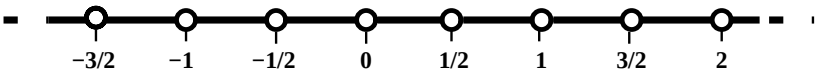
Feedback If you find that you have forgotten some of the techniques in this section, then you can revise them using materials from earlier modules such as MST124 and MST125.

Sketching sets and curves

13. As

$$\{n/2 : n \in \mathbb{Z}\} = \{\ldots, -3/2, -1, -1/2, 0, 1/2, 1, 3/2, 5/2, \ldots\}$$

(all those numbers that are integer multiples of $1/2$), the set $\mathbb{R} - \{n/2 : n \in \mathbb{Z}\}$ consists of all those real numbers that are not integer multiples of $1/2$,



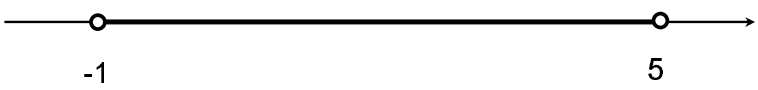
To find the set $\{x \in \mathbb{R} : |x - 2| < 3\}$, notice that $|x - 2| < 3$ if and only if

$$-3 < x - 2 < 3,$$

and this rearranges to give

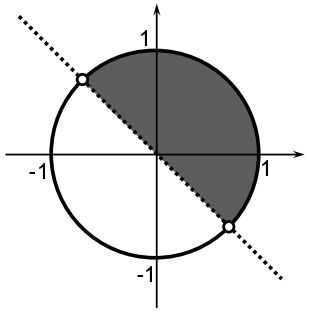
$$-1 < x < 5.$$

Hence the set is



You can think of $\{x \in \mathbb{R} : |x - 2| < 3\}$ as being the set of all points whose distance from 2 is less than 3.

14. The inequality $x + y > 0$ rearranges to give $y > -x$ and so this is the part of the plane lying *strictly* above the line $y = -x$. The inequality $x^2 + y^2 \leq 1$ determines the region of the plane lying within the circle with centre the origin and radius 1 and includes the circle itself. The intersection of these two regions is illustrated in the following diagram. (Notice that the points where the line intersects the circle are not part of the region.)



15. (a) Here we notice that $x^2 + y^2 - 4x + 2y + 1 = 0$ can be written as $x^2 - 4x + y^2 + 2y + 1 = 0$, and then on completing the square for x and y, we obtain $(x-2)^2 - 4 + (y+1)^2 - 1 + 2 = 0$, which rearranges to give

$$(x-2)^2 + (y+1)^2 = 2^2;$$

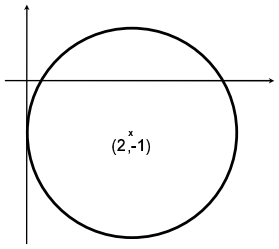
this is the equation of a circle, centre $(2, -1)$, radius 2.

(b) In this case we divide throughout by 36 to obtain

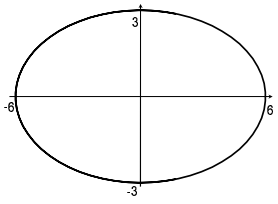
$$\frac{x^2}{6^2} + \frac{y^2}{3^2} = 1$$

which is the equation of an ellipse with centre $(0, 0)$ and with horizontal axis of length 12 and vertical axis of length 6.

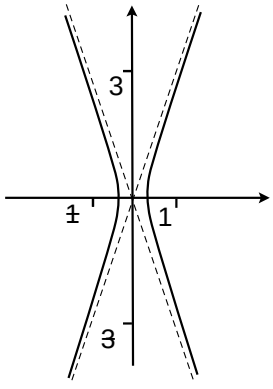
(c) This is the equation of a hyperbola symmetrical about the x- and y-axes.
Since $9x^2 - y^2 = (3x - y)(3x + y)$, the equations of its asymptotes are given by $y = 3x$ and $y = -3x$. When $y = 0$, we see that $x = \pm 1/3$.



(a) circle



(b) ellipse



(c) hyperbola

Feedback Once again, you should revise topics from earlier modules such as MST124 and MST125 if you got stuck with some of these questions.

Sequences and series

16. This is a geometric series, hence

$$\sum_{n=0}^{\infty} 5^{-n} = \frac{1}{1-\frac{1}{5}} = \frac{5}{4}.$$

17. (a) This series diverges: it is the harmonic series.

(b) Since the terms of the series, $\frac{(-1)^{n+1}}{n}$, alternate in sign and $\{1/n\}$ is a null sequence, we conclude by the alternating series test that this series converges.

(c) This is a standard example of a convergent series. It is shown in M337 that it converges to $\pi^2/6$.

(d) We observe that if we let $a_n = 5^n/(n!)$, then

$$\frac{a_{n+1}}{a_n} = \frac{\left(5^{n+1}/((n+1)!)\right)}{(5^n/(n!))} = \frac{5(n!)}{(n+1)\times(n!)} = \frac{5}{n+1} \to 0$$

as $n \to \infty$. Hence, by the ratio test for series, $\sum a_n$ converges.

(e) Since $n!/10^n \not\to 0$ as $n \to \infty$ it follows that the series diverges.

Feedback The only series you are assumed to have familiarity with in M337 are geometric series, which are covered, for example, in Unit 10 of MST124. However, experience of working with real series will help you to understand M337.

Differentiation and integration

18. (a) All the parts of this question use the formula $\frac{d}{dx}x^n = nx^{n-1}$, where $n \in \mathbb{R}$.

(i) $6x^2$ (ii) $2x^{-1/2} = \frac{2}{\sqrt{x}}$ (iii) $-5x^{-2} = -5/x^2$

(b) $-5\sin(1+5x)$

(c) $3x^2\sin(x) + x^3\cos(x)$

(d) $\frac{2-2\log_e(x)}{4x^2} = \frac{1-\log_e(x)}{2x^2}, \quad x > 0$

19. This is an application of the product rule for derivatives:

$$(uv)' = u'v + uv'$$

together with the chain rule:

$$(f \circ g)'(x) = g'(x)(f' \circ g)(x).$$

On letting $u(x) = e^{2x}$ and $v = \cos^2(x)$ we find from the chain rule that

$$u'(x) = 2e^{2x} \text{ and } v'(x) = -2\sin x\cos x.$$

Hence

$$\begin{aligned} f'(x) &= 2e^{2x}\cos^2 x - 2e^{2x}\sin x\cos x \\ &= 2e^{2x}\cos x(\cos x - \sin x). \end{aligned}$$

When $x = \pi/2$, we find $f'(x) = 0$.

20. (a) $\displaystyle\int_0^1 x^3\,dx = \left[\tfrac{1}{4}x^4\right]_{x=0}^{x=1} = \tfrac{1}{4}$

(b) On noting that $\frac{d}{dx}(x^2+1)^6 = 12x(x^2+1)^5$, we conclude that

$$\int 12x(x^2+1)\,dx = (x^2+1)^6 + C,$$

where C is an arbitrary constant.

(c) We use the partial fractions we found earlier to calculate

$$\begin{aligned}\int_4^5 \frac{1}{x(x-3)}\,dx &= \tfrac{1}{3}\int_4^5 \frac{1}{x-3} - \frac{1}{x}\,dx\\ &= \tfrac{1}{3}\left[\log_e(x-3) - \log_e x\right]_{x=4}^{x=5}\\ &= \tfrac{1}{3}\left[\log_e \frac{x-3}{x}\right]_{x=4}^{x=5}\\ &= \tfrac{1}{3}\left(\log_e \tfrac{2}{5} - \log_e \tfrac{1}{4}\right)\\ &= \tfrac{1}{3}\log_e \tfrac{8}{5}.\end{aligned}$$

(d) We use integration by parts, $\int fg' = fg - \int f'g$. Here we let $f(x) = x$ and $g'(x) = e^x$, and so $f'(x) = 1$ and $g(x) = e^x$. Thus

$$\begin{aligned}\int_0^1 xe^x\,dx &= [xe^x]_0^1 - \int_0^1 1\times e^x\,dx\\ &= e^1 - [e^x]_0^1\\ &= e^1 - (e^1 - 1) = 1.\end{aligned}$$

Feedback If it took you more than a few minutes to complete Question 18, then you should spend some time practising basic calculus before starting M337. Definite integrals frequently appear in the working of examples in M337, and you should be comfortable dealing with them. If the last few questions in this section took you a significant amount of time, then you should revise integration before starting M337.

Analysis

21. False. The function $f(x) = x$ satisfies $f'(x) = 1$ for all values of x, and the maximum value of f in $[0,1]$ is $f(1) = 1$.

22. False. The function f that satisfies $f(x) = -1$ for $x \le 0$ and $f(x) = 1$ for $x > 0$ does not have the required property. If f were a continuous function, then there would be such a point c, by the Intermediate Value Theorem.

23. True. If s_N denotes the Nth partial sum of the series, that is, $s_N = \sum_{n=1}^{N} a_n$, then $a_N = s_N - s_{N-1}$. If the series is convergent, then the sequence of partial sums s_N is also convergent, with limit s, say. (This is the definition of convergence for series.) Hence, by the combination rules for sequences,

$$a_N = s_N - s_{N-1} \to s - s = 0 \quad \text{as} \quad N \to \infty.$$

24. False. The converse says that if $a_n \to 0$ then $\sum a_n$ is convergent. This is false as, for example, the harmonic series $1 + 1/2 + 1/3 + 1/4 + \cdots$ is divergent.

25. (a) False. The implication does not work either way: $-1 < 0$ but $(-1)^2 > 0$, and $(-1)^2 < (-2)^2$ but $-1 > -2$.

(b) True. In this case, $(y - x) > 0$ if and only if $(y^2 - x^2) = (y - x)(y + x) > 0$. So $y > x$ if and only if $y^2 > x^2$.

Feedback for students who have completed M208
If you could do most of this section without difficulty, then you probably have the mathematical maturity to study M337. At the other extreme, if you missed the point of the questions or had difficulty following the solutions, then you are going to find M337 more difficult to follow – it may be wise to consider taking a different module. If you fall in between these situations, for example if you managed a couple of the questions fine by yourself and could follow all the solutions, then you are probably suitably prepared for M337, though you may find it challenging.

Feedback for students who have not completed M208
You may not have met all the techniques needed to answer some of these questions. If you cannot answer the questions and do not understand the solutions, then you may struggle with M337 – it may be sensible to consider a different module. If you have made a reasonable attempt at some of the questions, and you can understand the solutions, then, providing that you have successfully completed the questions in other sections, you probably have enough mathematical maturity for M337, although you may have to spend time catching up on a few topics before or at the start of the module.

M337 booklist

Since complex analysis is such a central topic in mathematics, many textbooks have been written about it. Most cover the same basic material, but at various levels of sophistication and with considerable variation in notation and terminology.

You should not need to consult other textbooks while studying M337, but in case you wish to do so, we give a few suggestions. We advise you to look carefully at any book before buying, as none have exactly the same approach and notation as M337, and student needs and levels of understanding vary. Note that these books often go out of stock or out of print for a while. Also, the prices of new and second hand maths texts may vary greatly from year to year. Libraries may well have older editions, which are just as useful.

We encourage you to post, on the M337 forum, how useful you have found any textbook while studying M337, so that other students can benefit from your experience.

Some of the books below are available as ebooks on the M337 Library Resources page. On that page you can also find links to various web applications for visualising complex functions.

Ian Stewart and David Tall: Complex Analysis (2008)

This book covers the same ground as the first three blocks of M337 and at a similar intellectual level. It is, perhaps, the book that most closely resembles M337 but with significant differences in notation, terms and methods. It is very readable but there are not many worked examples. There are quite a few exercises but no solutions. This could be used to complement the M337 materials by students wishing to get a broader understanding of complex analysis, or to see a different view of some of the concepts. It is not so suitable for students who simply wish to be able to apply the results. It assumes a sound knowledge of real analysis.

Alan Jeffrey: Complex Analysis and Applications (1991)

This is a large book which covers M337 up to the end of Unit C1 together with D1, and a lot more complex analysis that would be of interest to physicists and engineers. Although its order, approach and notation is different from M337, it should be reasonably straightforward for M337 students to dip into. It has many worked examples, lots of exercises, with reasonably full solutions to the odd numbered exercises. It could be useful to students with an interest in applying complex analysis elsewhere and to students looking for more examples.

Hilary Priestley: Introduction to Complex Analysis (2015)

A fairly concise abstract approach, roughly in the same order as M337 with some, but not too many, different terms and notations. It does not cover C3, D2 or D3 at all, though it is good on D1, conformal mappings. It has a few brief examples and exercises but no solutions. This book is more likely to be of interest to able students who like the abstract aspects of the module.

Tristan Needham: Visual Complex Analysis (1999)

This book emphasises the visual aspects of complex analysis of M337. It is an interesting and well-written book. The first two chapters give a good introduction to the subject. (An M337 student remarked that he has found these chapters an excellent preparation for M337.) A good book to read after M337 to further your knowledge and understanding of complex analysis.

James Brown and Ruel Churchill, Complex Variables and Applications (2013)

This book takes a similar approach to M337 but does not cover D2 and D3. It uses examples and exercise sets, with explanations of problem solving techniques.

Murray Spiegel: Complex Variables (1974)

This book (and updated versions of it with two co-authors) keeps the theory aspect of complex analysis to a minimum: it consists almost entirely of summaries of results, worked problems and exercises with brief answers. It covers most of the work up to the end of Unit D1 and is probably the book most used by past M337 students.

James Ogden and Emil Milewski: Complex Variables Problem Solver (1987)

This book consists entirely of problems with worked solutions. The solutions are easy to follow but they would not get full marks on M337, since we expect justification of any results used. However, it could be useful for students who feel they need to see more worked solutions. It has problems on most of the topics up to the end of Unit C1 and in Unit D1, and in many topics not relevant to M337.

Further reading

Here is a selection of books that build on complex analysis or develop the subject further, which you may like to read after the exam. The levels given here are only very approximate!

Michael Field & Martin Golubitsky: Symmetry in Chaos (1996)
Pre-university level: uses symmetry groups, complex numbers and iterated function systems to produce wonderful coloured images.

Benoit Mandelbrot: The Fractal Geometry of Nature (1982)
Pre-university level: the book that launched 'fractals' on the world.

David Mumford, Caroline Series, David Wright: Indra's Pearls: The Vision of Felix Klein (2015)
This describes computer explorations, using Möbius transformations, of delicate fractal objects which enjoy multiple symmetries arising from infinitely repeated reflections. Such objects were described by the great geometer Felix Klein in the late 1800s but the idea goes back to Indra's net of pearls in Buddhist mythology.

Lars Ahlfors: Complex Analysis (2013)
A classic text on complex analysis, originally written in 1953 - covering much of the material in M337 at a more sophisticated level, as well as topics such as infinite product representations of analytic functions.

Theodore Gamelin: Complex Analysis (2001)
A clearly written textbook on complex analysis, which covers most of the material of M337 and goes well beyond into many interesting complex analysis topics, some more suited to postgraduate study - with a wealth of exercises, many with outline solutions.

Kenneth Stephenson: Introduction to Circle Packing (2005)
A beautifully illustrated book on a fascinating geometric theory for illuminating complex analysis using circle packing.

Complex analysis in MSc modules

Several of the Open University MSc modules in mathematics use complex analysis. Here are the texts for those modules.

Tom Apostol: Introduction to Analytic Number Theory (1998)
This is the set text for *Analytic number theory I* (M823) and *Analytic number theory II* (M829). It is about applying techniques from analysis (such as properties of analytic functions, residues and contour integration) to tackle problems in number theory, including a proof of the Prime Number Theorem.

John Dettman: Applied Complex Variables (1985)
This is the set text for *Applied complex variables* (M828): covers much of the theory of M337, before moving on to further topics including potential theory, differential equations and Fourier transforms.

Kenneth Falconer: Fractal Geometry: Mathematical Foundations and Applications (2014)
This is the set text for *Fractal geometry* (M835): covers fractal dimensions, fractals and their applications, including material on quadratic Julia sets.